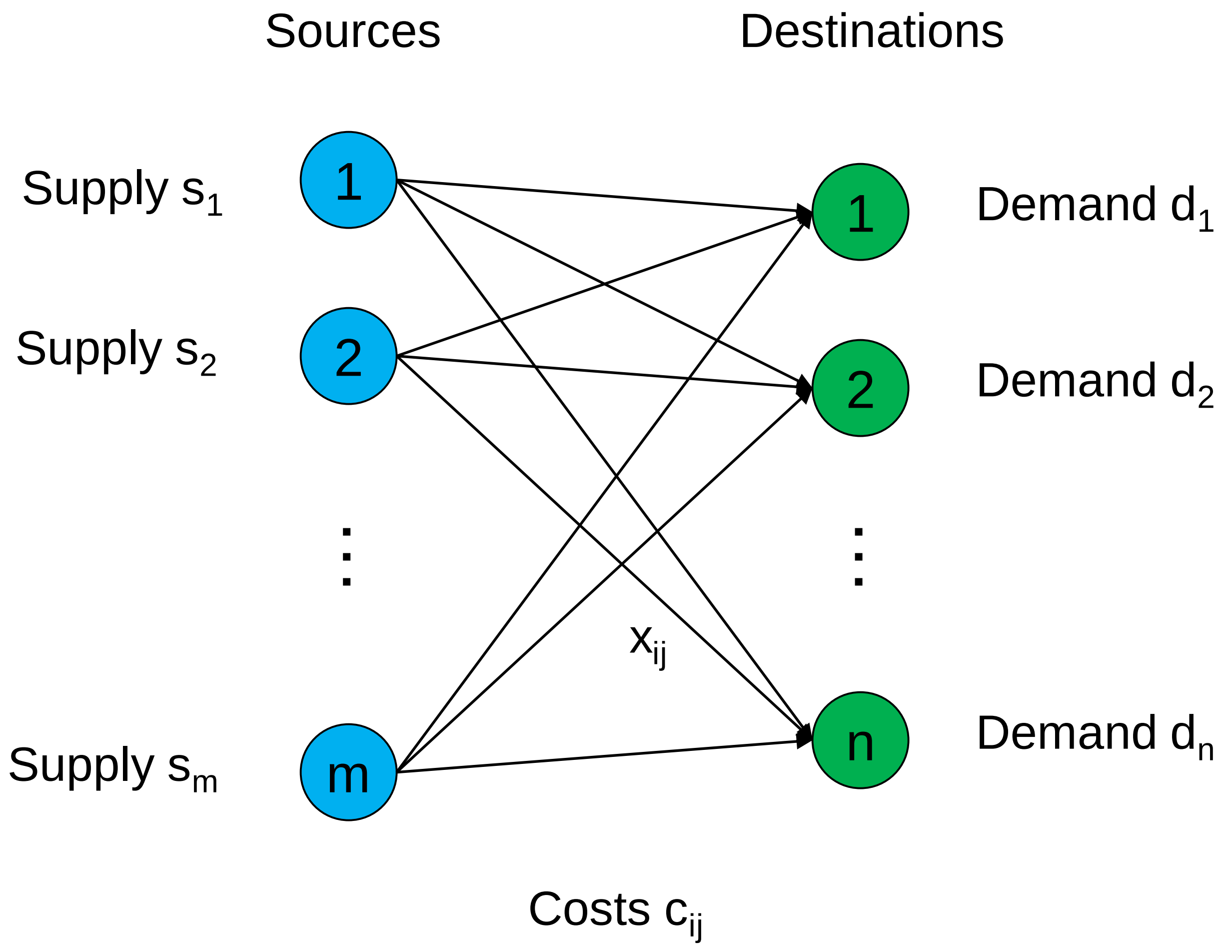
TRANSPORTATION PROBLEM

The Transportation Problem

- The problem of finding the **minimum-cost** distribution of a given commodity
from a group of supply centers (**sources**) $i=1,\ldots,m$
to a group of receiving centers (**destinations**) $j=1,\ldots,n$
- Each source has a certain supply (s_i)
- Each destination has a certain demand (d_j)
- The cost of shipping from a source to a destination is directly proportional to the number of units shipped

Simple Network Representation



Transportation Problem

LP Formulation:

The linear programming formulation in terms of the amount shipped from the sources to the destinations, x_{ij} , can be written as:

Min $\sum_{i}\sum_{j} c_{ij}x_{ij}$ *(total transportation cost)*

s.t. $\sum_{j} x_{ij} \leq s_i$ for each source i *(supply constraints)*

$\sum_{i} x_{ij} = d_j$ for each destination j *(demand constraints)*

$x_{ij} \geq 0$ for all i and j *(non-negativity constraints)*

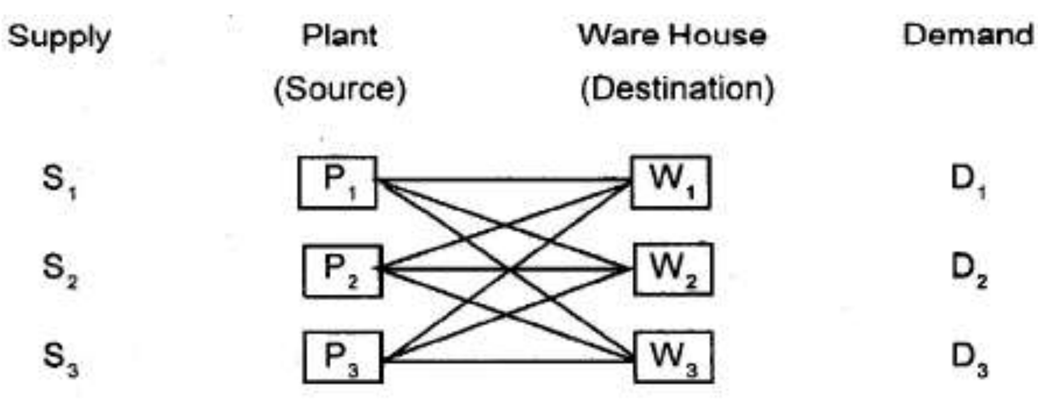
Note: Balanced problem has equations while non-balanced problem has inequalities

Example

Transportation as a typical situation shown in the manufacturer example

- Manufacturer has three plants P_1, P_2, P_3 producing same products.
- From these plants, the product is transported to three warehouses W_1, W_2 and W_3
- Each plant has a limited capacity, and each warehouse has specific demand. Each plant transport to each warehouse, but transportation cost vary for different combinations.

❖ The problem is to determine the quantity for each warehouse in order to minimize total transportation costs.



The Transportation Table

Source	Destination 1	2	…	n	Supply
1	c_{11}	c_{12}	…	c_{1n}	s_1
2	c_{21}	c_{22}	…	c_{2n}	s_2
…	…	…	…	…	…
m	c_{m1}	c_{m2}	…	c_{mn}	s_m
Demand	d_1	d_2	…	d_n	

Steps in transportation problem solution

1. Identifying a basic feasible solution

- Northwest corner rule
- Least cost method
- Vogel's approximation

2. Obtaining optimal solution

- Stepping stone method
- MODI (Modified distribution method)

Finding an Initial Feasible Solution

There are a number of methods for generating an initial feasible solution for a transportation problem.

Consider three of the following

(i) North West Corner Method

(ii) Least Cost Method

(iii) Vogel's Approximation Method

Example-1

		Ware houses			Supply S_i
		W_1	W_2	W_3	
	P_1	7	6	9	20
Plants	P_2	5	7	3	28
	P_3	4	5	8	17
Demand	D_j	21	25	19	65

Solution by NWCM

		WARE HOUSES			SUPPLY
		W_1	W_2	W_3	S_i
	P_1	7 20	6	9	~~20~~ 0
	P_2	5 1	7 25	3 2	~~28~~ ~~27~~ ~~2~~ 0
PLANTS	P_3	4	5	8 17	17
	D_j	~~21~~ ~~1~~ 0	~~25~~ 0	~~19~~ ~~17~~ 0	65

Total Cost = 20x7 +1 X 5 + 25 x 7 + 2 x 3 + 17 x 8

= Rs. 462

Solution by Least cost Method

	Ware House			Supply
	W_1	W_2	W_3	S_i
P_1	$\boxed{7}$	$\boxed{6}$ 20	$\boxed{9}$	20 0
P_2	$\boxed{5}$ 4	$\boxed{7}$ 5	$\boxed{3}$ 19	~~28~~ ~~9~~ ~~5~~ 0
P_3	$\boxed{4}$ 17	$\boxed{5}$	$\boxed{8}$	~~17~~ 0
Demand D_j	~~21~~ ~~4~~ 0	~~25~~ ~~5~~ 0	~~19~~ 0	65

Total cost = 20 x 6 + x 5 + 5 x 7 + 19 x 3 + 17 x 4 = Rs. 300

This total cost is less than the total cost obtained by NWCM.

Solution by Vogel's Approx Method

Warehouse

	W_1	W_2	W_3	S_i
P_1	7	6 20	9	~~20~~ 0
P_2	5 9	7	3 19	~~28~~ ~~9~~ 0
P_3	4 12	5 5	8	~~17~~ ~~5~~ 0
D_j	~~21~~ ~~12~~ 0	~~25~~ ~~20~~ 0	~~19~~ 0	65

Column penalties

1	1	5
1	1	-
3	1	-

Row penalities

1	1	1
2	2	-
1	1	1

The total transportation cost associated with this method is

Total cost = 20 x 6 + 9 x 5 + 19 x 3 + 12 x 4 + 5 x 5 = Rs. 295.

Example-2

Find the total transportation cost by

i. North west corner method

ii. Least cost method

iii. Vogel's approximation method

Sources	A	B	C	D	Supply
	3	1	7	4	300
	2	6	5	9	400
	8	3	3	2	500
Demand	250	350	400	200	

(Columns A–D: Destinations)

Finding the Optimal Solution

Once an initial solution has been found, the next step is to test that solution for optimality. The following two methods are widely used for testing the solutions:

- Stepping Stone Method
- Modified Distribution Method

The two methods differ in their computational approach but give exactly the same results and use the same testing procedure.

Stepping-Stone Method

In this method we calculate the net cost change that can be obtained by introducing any of the unoccupied cells into the solution.

Steps followed:

1. Make sure that the number of occupied cells is exactly equal to m+n-1, where m=number of rows and n=number of columns.
2. Evaluate each unoccupied cells by following its closed path and determine its net cost change.
3. Determine the quantity to be shipped to the selected unoccupied cell. Trace the closed path for the unoccupied cell and identify the minimum quantity by considering the minus sign in the closed path.

Modified Distribution (MODI) Method

The MODI method is a more efficient procedure of evaluating the unoccupied cells. The modified transportation table of the initial solution is shown below

	W_1	W_2	W_3	S_i	U_i
P_1	7 +2	6 20	9 +6	20	$U_1 = 0$
P_2	5 9	7 +1	3 19	28	$U_2 = 0$
P_3	4 12	5 5	8 +6	17	$U_3 = -1$
D_j	21	25	19	65	
V_j	$V_1=5$	$V_2 = 6$	$V_3=3$		

Special Cases in Transportation Problem

- Multiple Optimal Solutions
- Unbalanced Transportation Problems
- Degeneracy in Transportation Problem